VECTOR DERIVATIVES

Cartesian Coordinates (x,y,z)

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$\vec{\nabla}V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \hat{y}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \hat{z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

$$\vec{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (r,f,z)

$$\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + A_z\hat{z}$$

$$\vec{\nabla}V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{r}\left[\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right] + \hat{\phi}\left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right] + \hat{z}\frac{1}{r}\left[\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right]$$

$$\vec{\nabla}^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (r,q,f)

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

$$\vec{\nabla}V = \hat{r}\frac{\partial V}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla}\times\vec{A} = \hat{r}\frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi\sin\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(rA_\phi)}{\partial r}\right] + \hat{\phi}\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]$$

$$\vec{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

COORDINATE TRANSFORMATIONS

From Rectangular (x,y,z) to Cylindrical (r,f,z) and Cylindrical to Rectangular

$$\hat{r}(\phi) = \cos(\phi)\hat{x} + \sin(\phi)\hat{y}$$
$$\hat{\phi}(\phi) = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$$
$$\hat{z} = \hat{z}$$
$$r = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

$$\hat{x} = \cos(\phi)\hat{r}(\phi) - \sin(\phi)\hat{\phi}(\phi)$$
$$\hat{y} = \sin(\phi)\hat{r}(\phi) + \cos(\phi)\hat{\phi}(\phi)$$
$$\hat{z} = \hat{z}$$
$$x = r\cos(\phi)$$
$$y = r\sin(\phi)$$
$$z = z$$

From Rectangular (x,y,z) to Spherical (r,q,f) and Spherical to Rectangular

$$\hat{r}(\theta,\phi) = \sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\theta)\hat{z}$$
$$\hat{\theta}(\theta,\phi) = \cos(\theta)\cos(\phi)\hat{x} + \cos(\theta)\sin(\phi)\hat{y} - \sin(\theta)\hat{z}$$
$$\hat{\phi}(\phi) = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$$
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\hat{x} = \sin(\theta)\cos(\phi)\hat{r}(\theta,\phi) + \cos(\theta)\cos(\phi)\hat{\theta}(\theta,\phi) - \sin(\phi)\hat{\phi}(\phi)$$
$$\hat{y} = \sin(\theta)\sin(\phi)\hat{r}(\theta,\phi) + \cos(\theta)\sin(\phi)\hat{\theta}(\theta,\phi) + \cos(\phi)\hat{\phi}(\phi)$$
$$\hat{z} = \cos(\theta)\hat{r}(\theta,\phi) - \sin(\theta)\hat{\theta}(\theta,\phi)$$
$$x = r\sin(\theta)\cos(\phi)$$
$$y = r\sin(\theta)\sin(\phi)$$
$$z = r\cos(\theta)$$
$$\sqrt{x^2 + y^2} = r\sin(\phi)$$

From Cylindrical (r_c,f,z) to Spherical (r_s,q,f) and Spherical to Cylindrical

$$\hat{r}_s(\theta,\phi) = \sin(\theta)\hat{r}_c(\phi) + \cos(\theta)\hat{z}$$
$$\hat{\theta}(\theta,\phi) = \cos(\theta)\hat{r}_c(\phi) - \sin(\theta)\hat{z}$$
$$\hat{\phi}(\phi) = \hat{\phi}(\phi)$$
$$r_s = \sqrt{r_c^2 + z^2}$$
$$\theta = \tan^{-1}\left(\frac{r_c}{z}\right)$$
$$\phi = \phi$$

$$\hat{r}_c(\phi) = \sin(\theta)\hat{r}_s(\theta,\phi) + \cos(\theta)\hat{\theta}(\theta,\phi)$$
$$\hat{\phi}(\phi) = \hat{\phi}(\phi)$$
$$\hat{z} = \cos(\theta)\hat{r}_s(\theta,\phi) - \sin(\theta)\hat{\theta}(\theta,\phi)$$
$$r_c = r_s\sin(\theta)$$
$$\phi = \phi$$
$$z = r_s\cos(\theta)$$

ELECTROSTATIC EQUATION LIST

$$\vec{F}_2 = q_2 \vec{E}_2$$

$$\vec{E}_2 = \frac{q_1}{4\pi\varepsilon_o R_{12}^2}\hat{r}_{12}$$

$$\varepsilon_0 = \frac{1}{36\pi}x10^{-9}$$

$$V = -\int \vec{E}\bullet d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V$$

$$\oiint \vec{D}\bullet d\vec{S} = q_{enc}$$

$$W = qV$$

$$\vec{\nabla}^2 V = -\frac{\rho}{\varepsilon}$$

$$dq = \rho_l dl = \rho_s dS = \rho_V dV$$

$$V = \frac{q}{4\pi\varepsilon_o R}$$

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P}$$

$$\vec{\nabla}\bullet\vec{D} = \rho_v$$

$$\vec{\nabla}\bullet\vec{P} = -\rho_{bv}$$

$$C = \frac{q}{V}$$

$$R = \frac{V}{I}$$

$$\vec{J} = \sigma\vec{E}$$

$$\vec{J} = qn\vec{v}$$

$$W_e = \frac{1}{2}\iiint \vec{E}\bullet\vec{D}dV = \frac{1}{2}\iiint \rho V dV$$

$$\vec{F}_q = -\vec{\nabla}W$$

$$I = \iint \vec{J}\bullet d\vec{S}$$

$$\oiint \vec{J}\bullet d\vec{S} = -\frac{dq}{dt}$$

$$\vec{\nabla}\bullet\vec{J} = -\frac{\partial\rho_v}{\partial t}$$

$$q = 16x10^{-19}$$

	$Cylindrical(r,\phi,z)$	$Spherical(r,\theta,\phi)$
$d\vec{l}$	$dr\hat{r} + rd\phi\hat{\phi} + dz\hat{z}$	$dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$
$d\vec{S}$	$rd\phi dz\hat{r}$; $drdz\hat{\phi}$; $rdrd\phi\hat{z}$	$r^2\sin\theta d\theta d\phi\hat{r}$; $r\sin\theta drd\phi\hat{\theta}$; $rdrd\theta\hat{\phi}$
dV	$rdrd\phi dz$	$r^2\sin\theta drd\theta d\phi$

MAGNETOSTATIC EQUATION LIST

$$Id\vec{L} = \vec{K}dS = \vec{J}dV$$

$$\vec{B}_2(\vec{r}_2) = \frac{\mu}{4\pi}\oint_{C_1}\frac{I_1(\vec{r}_1)d\vec{L}_1 \times \hat{r}_{21}}{r_{21}^2} = \frac{\mu}{4\pi}\iint_{S_1}\frac{\vec{K}_1(\vec{r}_1)dS_1 \times \hat{r}_{21}}{r_{21}^2} = \frac{\mu}{4\pi}\iiint_{V_1}\frac{\vec{J}_1(\vec{r}_1)dV_1 \times \hat{r}_{21}}{r_{21}^2}$$

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\oint_C \vec{H} \cdot d\vec{L} = I_{enc}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{A}(\vec{r}_2) = \frac{\mu}{4\pi}\iiint_{V_1}\frac{\vec{J}_1(\vec{r}_1)dV_1}{r_{21}}$$

$$\vec{\nabla}^2\vec{A}(\vec{r}_2) = -\mu\vec{J}(\vec{r}_2)$$

$$\vec{J}_b(\vec{r}) = \vec{\nabla} \times \vec{M}$$

$$\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right) = \mu_o\mu_r\vec{H}$$

$$\vec{M} = \chi_m\vec{H}$$

$$\Phi_{12} = \iint_S \vec{B}_1 \cdot d\vec{S}_2$$

$$L = \frac{\Psi}{I}$$

$$M_{12} = \frac{\Psi_{12}}{I_1}$$

$$d\vec{m} = Id\vec{S}$$

$$\vec{A} = \frac{\mu\vec{m} \times \hat{r}}{4\pi r^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu m}{4\pi r^3}\left[2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right]$$

$$W_m = \frac{1}{2}\iiint_V \vec{B} \cdot \vec{H}\, dV$$

ELECTRODYNAMIC FIELD RELATIONS

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \vec{\nabla} \cdot \vec{D} = \rho \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\gamma = j\omega\sqrt{\varepsilon\mu}\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}} \qquad \sqrt{j} = \pm\frac{\sqrt{2}}{2}(1 + j)$$

$$(1 + x)^{1/2} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$$

$$(1 + x)^{-1/2} \approx 1 - \frac{1}{2}x + \frac{3}{8}x^2$$

$$SWR = \frac{1 + |\rho|}{1 - |\rho|} \qquad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \qquad \tan\theta = \frac{|\vec{J}_c|}{|\vec{J}_d|} = \frac{\sigma}{\omega\varepsilon}$$

$$\vec{E}(\vec{r}) = \vec{E}_0^a(\vec{r})e^{-\gamma\vec{r}\cdot\hat{n}} + \vec{E}_0^b(\vec{r})e^{+\gamma\vec{r}\cdot\hat{n}}$$

$$E(z) = E_o^a e^{-\gamma z} + E_o^b e^{+\gamma z}$$

$$k_i \sin\vartheta_i = k_r \sin\vartheta_r = k_t \sin\vartheta_t$$

$$\eta(d) = \eta_1\left[\frac{\eta_2\cos(\beta_1 d) + j\eta_1\sin(\beta_1 d)}{\eta_1\cos(\beta_1 d) + j\eta_2\sin(\beta_1 d)}\right] = \eta_1\left[\frac{\eta_2 + j\eta_1\tan(\beta_1 d)}{\eta_1 + j\eta_2\tan(\beta_1 d)}\right]$$

$$Z(z) = \eta_1\left[\frac{\eta_2\cos(\beta_1 d) + j\eta_1\sin(\beta_1 d)}{\eta_1\cos(\beta_1 d) + j\eta_2\sin(\beta_1 d)}\right] \text{ or } \eta_1\left[\frac{\eta_2\cosh(\gamma_1 d) + \eta_1\sinh(\gamma_1 d)}{\eta_1\cosh(\gamma_1 d) + \eta_2\sinh(\gamma_1 d)}\right]$$

$$\tilde{\rho}(z) = \frac{E_o^-}{E_o^+}e^{+2\gamma z}$$

$$\varepsilon_0 = 8.854x10^{-12} \qquad \mu_0 = 4\pi x10^{-7}$$

$$\oiint_s (\vec{E} \times \vec{H}) \cdot d\vec{S} = -\frac{\partial}{\partial t}\iiint_v \left[\frac{1}{2}\vec{D} \cdot \vec{E} + \frac{1}{2}\vec{B} \cdot \vec{H}\right] dV - \iiint_v \vec{J} \cdot \vec{E}\, dV$$

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} \qquad \oint_c \vec{H} \cdot d\vec{l} = \iint_s \left[\vec{J} + \frac{d\vec{D}}{dt}\right] \cdot d\vec{S}$$

TRANSMISSION LINE RELATIONS

$$V(x) = V_o^+ e^{-\gamma x} + V_o^- e^{+\gamma x}$$

$$I(x) = I_o^+ e^{-\gamma x} + I_o^- e^{+\gamma x}$$

$$Z(x) = Z(d) = Z_o \left[\frac{Z_L \cosh(\gamma d) + Z_o \sinh(\gamma d)}{Z_o \cosh(\gamma d) + Z_L \sinh(\gamma d)} \right] = Z_o \left[\frac{e^{+\gamma d} + \rho_L e^{-\gamma d}}{e^{+\gamma d} - \rho_L e^{-\gamma d}} \right]$$

$$Z_o = \sqrt{\frac{\left(j\omega\overline{L} + \overline{R}\right)}{\left(j\omega\overline{C} + \overline{G}\right)}}; \quad \gamma = \sqrt{\left(j\omega\overline{L} + \overline{R}\right)\left(j\omega\overline{C} + \overline{G}\right)}$$

$$SWR = \frac{1 + |\rho_L|}{1 - |\rho_L|}$$

$$\tilde{\rho}(d) = \rho_L e^{-2\gamma d}$$

where

$$d = L - x$$

$$e^{+\gamma x} = e^{+\alpha x}\left[\cos(\beta d) + j \sin(\beta d)\right]$$

$$\rho_L = \left[\frac{Z_L - Z_o}{Z_L + Z_o} \right]$$

$$\sinh(j\beta d) = j \sin(\beta d)$$

$$\cosh(j\beta d) = \cos(\beta d)$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\sin 2\alpha = 2 \sin\alpha \cos\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$1 = \cos^2\alpha + \sin^2\alpha$$

FIELDS FOR RECTANGULAR WAVEGUIDE

TM_{mn}

$$E_z(\vec{r}) = E_{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$E_x(\vec{r}) = -\frac{j\beta_{mn}m\pi}{ak_{cmn}^2}E_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$E_y(\vec{r}) = -\frac{j\beta_{mn}n\pi}{bk_{cmn}^2}E_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$H_x(\vec{r}) = \frac{j\omega\varepsilon n\pi}{bk_{cmn}^2}E_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$H_y(\vec{r}) = -\frac{j\omega\varepsilon m\pi}{ak_{cmn}^2}E_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$\beta_{mn} = \sqrt{\varepsilon\mu\omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

TE_{mn}

$$H_z(\vec{r}) = H_{mn}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$E_x(\vec{r}) = \frac{j\omega\mu n\pi}{bk_{cmn}^2}H_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$E_y(\vec{r}) = -\frac{j\omega\mu m\pi}{ak_{cmn}^2}H_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$H_x(\vec{r}) = \frac{j\beta_{mn}m\pi}{ak_{cmn}^2}H_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$H_y(\vec{r}) = \frac{j\beta_{mn}n\pi}{bk_{cmn}^2}H_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z}$$

$$\beta_{mn} = \sqrt{\varepsilon\mu\omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

NOTE! The β_{mn} *and* k_{cmn} *for the TE and TM modes are not necessarily the same where m and n are appropriately chosen integers.*

COORDINATES AND VECTOR OPERATIONS

Coordinate \ Metric (u_1,u_2,u_3) \ (h_1,h_2,h_3)	h_1	h_2	h_3
Cartesian (x,y,z)	1	1	1
Cylindrical (ρ,ϕ,z) or (r,ϕ,z)	1	ρ or r	1
Spherical (r,θ,ϕ)	1	r	r $\sin\theta$

$$dV = h_1 h_2 h_3\, du_1\, du_2\, du_3$$

$$d\vec{l} = h_1 du_1 \hat{u}_1 + h_2 du_2 \hat{u}_2 + h_3 du_3 \hat{u}_3$$

$$d\vec{S} = h_2 h_3 du_2 du_3 \hat{u}_1 + h_1 h_3 du_1 du_3 \hat{u}_2 + h_1 h_2 du_1 du_2 \hat{u}_3$$

$$\vec{\nabla}\Phi = \hat{u}_1 \frac{1}{h_1}\frac{\partial\Phi}{\partial u_1} + \hat{u}_2 \frac{1}{h_2}\frac{\partial\Phi}{\partial u_2} + \hat{u}_3 \frac{1}{h_3}\frac{\partial\Phi}{\partial u_3}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial\left(h_2 h_3 E_1\right)}{\partial u_1} + \frac{\partial\left(h_1 h_3 E_2\right)}{\partial u_2} + \frac{\partial\left(h_1 h_2 E_3\right)}{\partial u_3}\right]$$

$$\vec{\nabla}x\vec{E} = \frac{1}{h_2 h_3}\left[\frac{\partial\left(h_3 E_3\right)}{\partial u_2} - \frac{\partial\left(h_2 E_2\right)}{\partial u_3}\right]\hat{u}_1 + \frac{1}{h_1 h_3}\left[\frac{\partial\left(h_1 E_1\right)}{\partial u_3} - \frac{\partial\left(h_3 E_3\right)}{\partial u_1}\right]\hat{u}_2 + \frac{1}{h_1 h_2}\left[\frac{\partial\left(h_2 E_2\right)}{\partial u_1} - \frac{\partial\left(h_1 E_1\right)}{\partial u_2}\right]\hat{u}_3$$

$$\vec{\nabla}^2\Phi = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial\Phi}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial\Phi}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial\Phi}{\partial u_3}\right)\right]$$

VECTOR IDENTITIES & TRIGNOMETRIC RELATIONS

$$\vec{A}\cdot\vec{B}\times\vec{C}=\vec{B}\cdot\vec{C}\times\vec{A}=\vec{C}\cdot\vec{A}\times\vec{B}$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}\cdot\vec{C}\right)-\vec{C}\left(\vec{A}\cdot\vec{B}\right)$$

$$\vec{\nabla}\left(\psi V\right)=\psi\vec{\nabla}V+V\vec{\nabla}\psi$$

$$\vec{\nabla}\cdot\left(\psi\vec{A}\right)=\psi\vec{\nabla}\cdot\vec{A}+\vec{A}\cdot\vec{\nabla}\psi$$

$$\vec{\nabla}\times\left(\psi\vec{A}\right)=\psi\vec{\nabla}\times\vec{A}+\vec{\nabla}\psi\times\vec{A}$$

$$\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right)=\vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right)$$

$$\vec{\nabla}\cdot\vec{\nabla}V=\vec{\nabla}^2V$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{A}=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)-\vec{\nabla}^2\vec{A}$$

$$\vec{\nabla}\times\vec{\nabla}V=0$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right)=0$$

$$\iiint_V \vec{\nabla}\cdot\vec{A}\,dV=\oiint_S \vec{A}\cdot dS$$

$$\iint_S \left(\vec{\nabla}\times\vec{A}\right)\cdot dS=\oint_C \vec{A}\cdot d\vec{l}$$

$$\cos 2\alpha=1-2\sin^2\alpha=2\cos^2\alpha-1$$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$1=\cos^2\alpha+\sin^2\alpha$$