Optimization of a Five-Cell Niobium Cavity

Myong Holl[a], Mohamed Trabia[b], and Robert A. Schill, Jr.[c] *University of Nevada Las Vegas*
[a,b] Department of Mechanical Engineering
[c] Department of Electrical Engineering
4505 Maryland Parkway
Las Vegas, Nevada 89154-4026
[b] mbt@me.unlv.edu
[c] schill@ee.unlv.edu

Abstract - *One means of handling nuclear spent fuel is to transmute the waste with the aid of an accelerator. In this approach, a particle accelerator produces protons that react with a heavy metal target to produce neutrons. A major component of the system is a linear accelerator that can accelerate protons to GeV energies. Los Alamos National Laboratory (LANL) has been developing a superconducting rf (SCRF) high-current linear accelerator for this purpose. One of the major components of SCRF is the multiple-cell niobium cavity. A localized resonant process known as multipacting has been a concern in the design of such cavities. Multipacting absorbs the rf power such that it becomes difficult to increase the cavity field strength when increasing the input power. At the University of Nevada Las Vegas, an optimization procedure using commercial field modeling codes developed by Field Precision Inc. (MESH, WAVSIM) coupled with a fuzzy logic optimization code and various controlling codes written in MATLAB has been developed for multipacting studies. Currently, the fully automated system of codes modifies a five-cell niobium seed cavity geometry until the desired mode and resonant frequency of the cavity is obtained. Results are compared with LANL's existing five-cell niobium cavity.*

I. INTRODUCTION

Accelerator transmutation of waste is one approach to deal with spent nuclear fuel. In this approach, a particle accelerator produces protons that react with a heavy metal target to produce neutrons. These neutrons are used to transmute long-lived radioactive isotopes into shorter-lived isotopes that are easier to be handled. Los Alamos National Laboratory (LANL) has been an active participant in developing a superconducting rf high-current linear accelerator for the accelerator production of tritium (APT) program.[1-3] The superconducting rf section of the linear accelerator has three major components: niobium cavities, power couplers, and cryomodules.

Niobium cavities are important parts of the integrated high-power superconducting linac. One major concern in the design of niobium cavities is to minimize multipacting losses. The shape of the cavity structure, its material composition, and the surface treatment impact the mitigation of the multipacting processes.[4-6] It has been shown that cavities with a spherical wall geometry along the cavity axis inhibits multipacting relative to elliptical cavities.[4] Researchers in several countries have tested various cavity shapes. They concluded that elliptically shaped cells are the most appropriate shapes for superconducting cavities due to their ability to be mechanically tuned, mechanically stable, and easily drained during cavity preparation. Subtle changes in a cavity design may significantly influence multipacting.[5, 7-9]

While models have been suggested for minimizing multipacting, a practical means of manufacturing the cavity walls to obtain optimal designs are still an issue. Attempting to improve the performance of multiple niobium cavities may be a daunting task because of the computational load associated with the evaluation of a particular design and the large number of variables and constraints involved. The following three levels of optimization in the order of importance must be carefully balanced in design: primary rf parameters (design frequency, high Q-factor, minimize field emission and multipacting, and maximize peak field ratios), mechanical properties (thickness of material, stiffness under vacuum load, tunability, and stability against Lorentz force detuning), and fabrication issues (choice of materials, joint properties, and ability to clean structure).[10] Using principles of nonlinear programming, a systematic method has been used to design multi-cell cavities based on the desired resonant frequency and mode of operation. Using MATLAB as the controlling environment, a set of commercial codes developed by Field Precision Inc. (MESH and WAVESIM)[11] were incorporated with a fuzzy simplex optimization algorithm[12] to search for an optimal shape for the multi-cell cavity. Once an optimum multi-cell cavity shape is identified, a multipactor code, Trak RF, is employed to examine the multipacting properties of the cavity.[6, 13, 14] This effort reports on the optimization of the cavity without taking into consideration multipacting results.

LANL has tested four β=64 five-cell APT niobium cavities designed for a π-mode, 700 MHz resonant frequency. The end cells of the cavity were slightly modified to attain the desired field flatness along the entire cavity and also to provide adequate coupling for the beam current.[1]

In this paper, the design of the end cells of the APT cavity is varied to verify the accuracy of the simulation software and possibly refine the existing design. Alternate designs are also pursued yielding the same target frequency. While the geometry of the end cell can be described using eleven parameters, a preliminary assessment of the problem identified three variables to which the design is fairly sensitive. Because of the complexity of the problem, only these three variables were used to move the cavity design closer to the target resonant frequency. To search for alternative geometries that can produce the target resonant frequency, randomly generated end cell geometries that satisfy the constraints of the problem were generated. The randomly generated cases of seed geometries were then optimized. Comparisons to the existing geometry are made leading to insights on how to design the end cell cavities of a five-cell cavity structure.

The paper is organized in the following manner. Section II describes the numerical calculation of the resonant frequency and cavity mode for LANL's APT cavity. The optimization technique based on the target frequency and mode for LANL's APT five-cell cavity geometry is discussed in Section III. Section IV describes the search for alternative end-cell designs. Conclusions are presented in the Section V.

II. CALCULATION OF THE CAVITY MODE AND RESONANT FREQUENCY

The optimization search is performed within a MATLAB environment. The optimization program uses two programs to calculate the objective function:

1. A mesh generator (MESH 4.5) which creates a conformal triangular mesh that corresponds to the evaluated geometry. Figure 1 shows a sample mesh of the cavity.
2. A frequency-domain finite-element electromagnetics field solution code (WAVESIM 5.0) uses the mesh to calculate the resonant frequency of the cavity.

Through a set of MATLAB scripts, these programs are activated in the DOS mode.
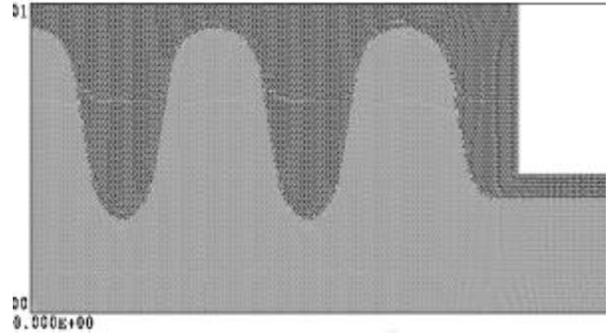

Fig. 1. Mesh of LANL's Beta-64 Five-Cell APT Niobium Cavity

MATLAB controls the input file for the MESH program. Initially, it defines a finer mesh concentration near the ellipse-like geometries relative to other regions of the cavity. The elliptical shapes are approximated by smaller line segments compared to the straight-line regions on the cavity surface. Based on the success/error codes from the MESH program, the MATLAB controlling code adjusts the mesh inputs to successfully produce a mesh that is well defined (not too large for the boundary) and allows WAVESIM to converge in a timely manner (not too fine). For example, when there are inverted elements in the mesh, the MATLAB code finds the coordinates of these inverted elements and concentrates an area around these coordinates to correct the inversion. If the solution region is too small, the MATLAB controlling code enlarges it. The "smoothing" option in MESH is used to make the transition between regions composed of larger and smaller elements. If the difference is too large, MESH will give an error. In this case, the MATLAB controlling code will define two buffer regions between those regions with large differences in element size. These buffer regions are currently weighted averages between the two regions of differing element size. To minimize the time it takes to run WAVESIM, the MATLAB controlling code attempts to limit the number of logical elements to 360,000.
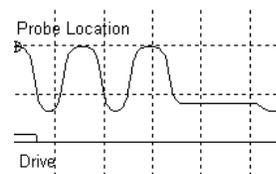

Fig. 2. Location of the Capacitive Drive Source and the Inductive Probe within the Five-Cell Cavity

WAVESIM searches for the resonant mode within a frequency range specified *a priori*. Using Field Precision's notation, the H wave solution for the open resonator is sought (H_ϕ, E_r, E_z). To excite the proper mode of interest, a small region on the cavity's axis near

the location of the maximum longitudinal electric field is set as the "drive" region with source current density of zero phase in the z direction. This region simulates the property of a small capacitive coupler exciting the electric field component of interest. An inductive probe is located near the drive point where H_ϕ is anticipated to be a maximum. The location of the capacitive coupler drive and the inductive probe are shown in Figure 2. Employing field symmetry about the cross section of the center cell, WAVESIM was constrained to finding only the $\pi/5$, $3\pi/5$ and π modes at the resonant frequency of interest. The MATLAB controlling code examined the sign of the fields in the each of the cavities guaranteeing that the desired π mode resonance was obtained.

III. OPTIMIZATION OF LANL'S APT CAVITY TO OBTAIN TARGET FREQUENCY

For a desired resonant frequency ω_T=700 MHz, the objective function of the optimization search is:

$$\text{minimize} \quad , \quad \left| \frac{\omega_T - \omega_0}{\omega_T} \right| \tag{1}$$

where, ω_0 is the angular frequency of the accelerating mode. Figure 3 illustrates the variables that describe the shape of the outer half of the end cells of the cavity. The cell is generated by rotating the profile of Figure 3 around its longitudinal axis. The profile is composed of four segments:
1. circular arc,
2. straight line,
3. portion of an ellipse that is tangent to the straight line of portion 2, and
4. straight line parallel to the cavity's longitudinal axis.

The total number of parameters that describe this geometry is eleven, which makes the optimization search challenging, since each evaluation of the resonant frequency proved to be a lengthy process. The parameters were tested to determine the sensitivity of the objective function (Equation (1)) to each of them. This test shows that the following three parameters are the most important ones: r_a, ϕ_a, and ϕ_L. These three variables are therefore used in the optimization search. Table I lists the values of the parameters that are fixed. The values of these parameters are equivalent to the corresponding ones of LANL's APT cavity. As shown in the following relations, the coordinates of the center of the ellipse are dependent variables,

$$x_e = x_{end} + a_e^2 \sqrt{\frac{1}{a_e^2 + \left(\frac{b_e^2}{\tan^2(f_L)} \right)}}$$

$$y_e = y_{end} + b_e \sqrt{1 - \frac{(x_{end} - x_e)^2}{a_e^2}} \tag{2}$$

where, x_{end} and y_{end} are the coordinates of the common point between the straight line segment and the ellipse.

TABLE I. Fixed Parameters of the End-Cell of the LANL's APT Cavity

Parameter	Value
y_a (cm)	16.339
L (cm)	5.814
a_e (cm)	2.425
b_e (cm)	9.700
L_o (cm)	15.081
y_o (cm)	8.000

The Fuzzy Simplex[12] optimization algorithm, which incorporates fuzzy logic to make the simplex search flexible, is employed. The problem is subject to the following constraints:

$$17.339 \ cm \le y_a + r_a \le 29.925 \ cm$$
$$10^o \le f_a \le 90^o$$
$$5^o \le f_L \le 20^o \tag{3}$$
$$y_o = 8 \ cm$$
$$3.5 \ cm \le x_e \le 14 \ cm$$

These constraints can be rearranged as:

$$1.00 \ cm \le r_a \le (29.925 - y_a) \ cm$$
$$10^o \le f_a \le 90^o$$
$$5^o \le f_L \le 20^o$$
$$\left(y_a + r_a - r_a \cos(f_a) - L\cos(f_L) + b_e \sqrt{1 - \frac{(x_{end} - x_e)^2}{a_e^2}} - b_e \right) = 8 \ cm$$
$$3.5 \ cm \le r_a \sin(f_a) + L\sin(f_L) + a_e^2 \sqrt{\frac{1}{a_e^2 + \left(\frac{b_e^2}{\tan^2(f_L)} \right)}} \le 14 \ cm \tag{4}$$

The geometry of the middle three cells are unchanged. The MATLAB controlling program shifts the cells in the y-direction to match the height of the end cell. The existing APT design[1,2] shown in Figure 4 is used as an initial guess. For this design, WAVESIM yielded a resonant frequency of 703.29 MHz (LANL computed a corresponding frequency of 699.90 MHz). After 39

function evaluations, the fuzzy simplex algorithm reaches the cell design of Figure 5, which has a 699.99 MHz resonant frequency. Table II shows the variable values at the initial guess and final result.
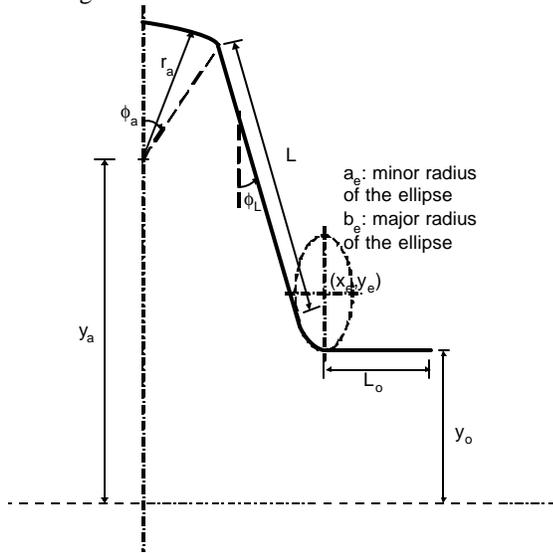

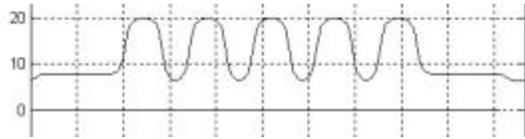Fig. 3. Variables for APT Cavity End Cell Geometry

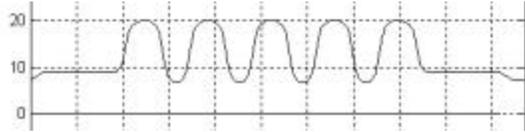Fig. 4. Initial Guess for the Optimization Search

Fig. 5. Final Result of the Optimization Search

TABLE II. Possible Designs for the End-Cell of the LANL's APT Cavity

	Initial Guess	Final Result
r_a (cm)	3.611	3.680
ϕ_a (rad.)	1.397	1.418
ϕ_L (rad.)	0.175	0.191
Resonant Frequency (MHz)	703.29	699.99

IV. SEARCH FOR ALTERNATE CELL DESIGNS

While the results of the previous section show the possibilities of tuning the existing design, it may be of interest to find alternative cavity designs. To explore this possibility, the three-parameter space was surveyed by generating random feasible points (random points that satisfies the geometric constraints of Equations 4). For each feasible point, WAVESIM calculated the π-mode resonant frequency. The variables for three cases found are shown in Table III. The corresponding geometry for Case # 3 is shown in Figure 6. These three cases were used as initial guesses for the fuzzy simplex algorithm. Of the three cases, Case #1 failed to converge. The results of these optimization searches are listed in Table IV. Figure 7 illustrates the optimized geometry of Case #3. The results show that other end cell geometries can yield the same resonant frequency while providing attractive features such as larger value of ϕ_L, which can improve drainage during chemical etching process as well as providing more structural stiffness.

TABLE III. Details of the Best Randomly Generated Feasible Points

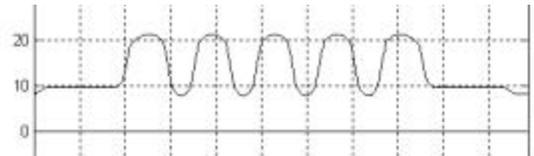|	Case #1	Case #2	Case #3
r_a (cm)	4.060	4.169	5.022
ϕ_a (rad.)	1.007	1.028	1.114
ϕ_L (rad.)	0.224	0.242	0.194
Resonant Frequency (MHz)	695.91	693.58	671.76

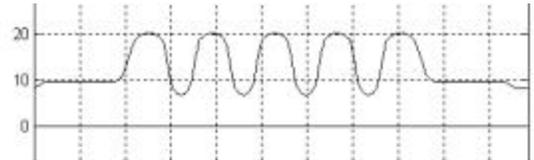Fig. 6. Cavity geometry for Case #3 of Table III

Fig. 7. Optimized cavity geometry for Case #3 of Table IV.

TABLE IV. Results of Optimization Search Using the Best Randomly Generated Feasible Points

	Case #1	Case #2	Case #3
r_a (cm)	X	4.086	3.985
ϕ_a (rad.)	X	1.019	1.278
ϕ_L (rad.)	X	0.293	0.315
Resonant Frequency (MHz)	Failed	699.16	699.60
Number of Function Evaluations	X	8	17

V. CONCLUSION

The hierarchy of cavity optimization must carefully balance the following three objectives in order of

importance: primary rf parameters, mechanical properties, and fabrication issues. An automated optimization process has been developed to provide alternative niobium cavity geometries based on a desired target resonant frequency and mode. Here, the optimization process was limited to altering the end cells of a five-cell niobium cavity design. Each of the end cell parameters was evaluated to determine the sensitivity of the objective function. Results show that the following three parameters are the most important ones: r_a, ϕ_a, and ϕ_L. These three variables are therefore used in the optimization search, which results in a cell geometry that produces the target frequency. To search for other different end cell geometries that produce the same target frequency, feasible random points were generated within the space of these three variables. Three of these cases were used as initial guesses. The optimization searches did not converge to the geometry of LANL's APT cavity. Results show that in addition to satisfying resonant frequency condition, the alternative geometries may have better mechanical and fabrication characteristics. Although the relative changes in some of the design may be small, other researchers have shown that small changes may significantly affect the multipacting process. At this time, multipacting has not been examined for these designs. Future multipacting studies will determine if these alternative designs are as robust compared to the existing APT cavity.

ACKNOWLEDGEMENTS

REFERENCES

1. K.C. DOMINIC CHAN, *Executive Summary: Development and Performance of Medium-Beta Superconducting Cavities*, Los Alamos National Laboratory Report No.#TPO-E41-R-TNS-X-00043, 2001.

2. T. TAJIMA, K.C.D. CHAN, et al, *Status of the LANL Activities in the Field of RF Superconductivity, Proceedings of the 10th Workshop on RF Superconductivity, Tsukuba, Japan, September 6-11, 2001 ,* Los Alamos National Laboratory Report No.# LA-UR-01-4988**,** (2001)

3. T. TAJIMA, K.C.D. CHAN, et al, *QO Degradation of LANL 700 MHz Beta=0.64 Elliptical Cavities and ANL 340 MHz Spoke Cavities, Eighth European Particle Accelerator Conference (EPAC2002), Paris France June 3-7, 2002, ,* Los Alamos National Laboratory Report No.**#** LA-UR-02-3080, (2002).

4. HASAN PADAMSEE, JENS KNOBLOCH, and TOM HAYS, *RF Superconductivity for Accelerators*, Chapt. 10, Wiley Interscience Publication, John Wiley, N.Y., (1998).

5. W. HARTUNG, F. KRAWCZYK, and H. PADAMSEE, *Studies of Multipacting in Axisymmetric Cavities for Medium-Velocity Beams, Presented at the Tenth Workshop on RF Superconductivity in Tsukuba, Japan,* Michigan State University Cyclotron Laboratory Report No.# MSUCL-1221, (2001).

6. F. KRAWCZYK, *Status of Multipacting Simulation Capabilities for SCRF Applications, Proceedings of the 10th Workshop on RF Superconductivity, Tsukuba, Japan, September 6-11, 2001 ,* Los Alamos National Laboratory Report No.# LA-UR-01-6447, (2001).

7. G. DEVANZ, "Multipactor Simulations in Superconducting Cavities and Power Couplers," *Physical Review Special Topics – Accelerators and Beams*, **4**, *1* p. 012001-1, (2001).

8. G. DEVANZ, *A 2D Multipactor Simulation Code for RF Components and Accelerating Cavities, Proceedings of EPAC 2000,Vienna, Austria,* p. 1366, (2000).

9. J. KNOBLOCK, W. HARTUNG and H. PADAMSEE, *Multipacting in 1.5 GHz Superconducting Niobium Cavities of the CEBAF Shape, Proceedings of the 8th Workshop on RF Superconductivity, Padova, Italy,* (1997).

10. F. KRAWCZYK, *private written communication through memorandum,* LANSCE-1:02-005, (1/18/02).

11. Field Precision Finite Element Software for Magnetic and Electric Field Analysis, http://www.fieldp.com/

12. M. TRABIA and X. LU, "A Fuzzy Adaptive Simplex Search Optimization Algorithm," *ASME Journal of Mechanical Design*, **123**, *2*, pp. 216, (2001).

13. S. HUMPHRIES JR., "Electron Multipactor Code for High-Power RF Window Development," *Particle Accelerators,* **62**, p. 139, (1999).

14. S. HUMPHRIES, JR. and D. REES, *Electron Multipactor Code for High-Power RF Devices, Proceedings of PAC 97, Vancouver, Canada,* p. 2428, (1998) [available online at http://www.JACoW.org/].